# Evolution Of Mathematical Vision From A Simple Problem

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## Abstract

*The students joining engineering have cleared the mathematics at standard* $12^{th}$. *Being a competitive exam the students are found to have prepared mathematics only from the scoring angles. This is surely required but along side the application of this mathematics is also equally important. The authors have noted this deficit on observing the results of G.T.U. Mathematics papers 1,2,3,4 and have come to remark to dedicate a few hours for the true word problems for the evolution of mathematical concepts. In this paper authors have attempted to extract as many as mathematical and geometrical models and attempted to convince the students the hidden mathematics.*

**Keywords:** Arithmetic mean, Geometric mean, Construction, Arc, Area, Rectangle, Square.

## 1. Introduction

In this paper, we are discussing of constructing a square under particular conditions. We would like to begin with a problem of construction of a square with given area, say $A$. The problem leads to the construction of length $x$ where $x^2 = A$ or $x = \sqrt{A}$. The following construction says that if the number $A$ is constructible then we can construct $\sqrt{A}$.

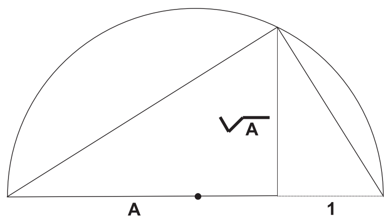


**Figure 1.**

But in case the the number $A$ is not constructible we can't say anything for example, the number $\pi$ is not constructible and $\sqrt{\pi}$ is also not constructible. This mean given a circle of unit radius, we can't construct a square with same area [1]. Now we address to the same problem by taking a rectangle and show that it is possible to construct the square with the same area.

## 2. Problem Definition

Given a rectangular sheet, transform it in to a square sheet by cut copy and past method only using a compass, straight unmarked scale, and geometrical constructions, so that there is no wastage of material.

## 3. Mathematical Modeling

At first look the problem seems to be a fairly simple problem in nature but modeling it provides a plentiful of mathematical concepts. We model the problem as follows.

Let $l$ and $b$ be the longer and shorter sides of the rectangular sheet. Let $A$ be the area of the rectangular sheet in square units. Thus

$$A = lb. \tag{1}$$

If the same area is achieved by a square of side $m$ then

$$A = m^2. \tag{2}$$

Thus equating (1) and (2)

$$m^2 = lb \quad \Rightarrow \quad m = \sqrt{lb}.$$

Thus very fundamentally this problem is basically a constructibility problem of two real positive numbers geometric mean. This problem can be further elaborated as follows.

[1]The approximate construction is given by Shrinivas Ramanujan.

**Case 1.** $l = kb, \quad k = n^2, \quad n \in \mathbb{N}.$

This yields the solution very immediately from simple modeling. (How!!!)

$A = lb = kbb = n^2b^2 = (nb)^2 = m^2$ where $m = nb$.

**Case 2.** $l = kb, \quad k \neq n^2, \quad l > b,$ and $l < 2b$. (for understanding construction easily)

**Step 1.** We create a square of side $b$ first. Yielding the area $A_1 = b^2$ and the remaining area $(l-b)b$. Construction is possible according to following figure.

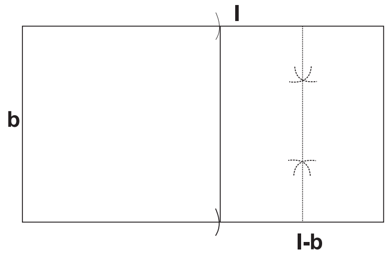


**Figure 2.**

**Step 2.** The remaining area $(l-b)b$ is cut in to four rectangular strips of equal area ( such is possible how!!!). And pasted on the four sides of the square as constructed earlier as shown in the figure.

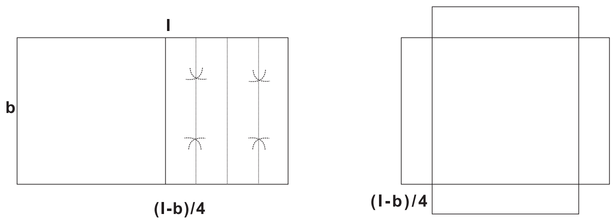


**Figure 3.**

**Step 3.** What does this yield? Identify the figure. This results in to a regular cross (Illustration: Red Cross Society Mark). This also yields the construction of arithmetic mean of the two numbers $l$ and $b$ (How!!!). Let the figure be named as vertices $A, B, C, D, E, F, G, , H, I, J, K, L$. Then,

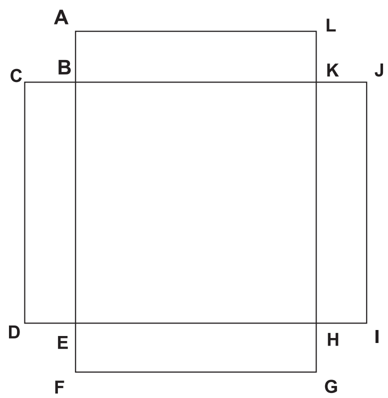


**Figure 4.**

$AF = \dfrac{l+b}{2} = b + 2\Big(\dfrac{l-b}{4}\Big)$. Thus the constructing a geometric mean provides an arithmetic mean as a by product.

**Step 4.** In this step, a cross is converted in to a square by cut and paste technique. Construct the four square of the size of $\dfrac{l-b}{4}$ from the center and paste it at four places as required. construction is possible. Is the constructed figure is square? The answer is no. Look the figure our objective is now to find the size of a strips from various sides and past them properly. So as to fill in the cut area. (Simple algebraic equations ).

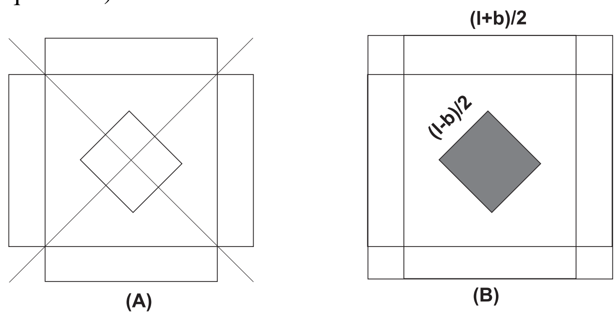


**Figure 5.**

The above four steps gives the how the square can be obtain by cut and paste method. In step 4 by simple algebraic equations we get the side of the square of length $\sqrt{lb}$. Let us verify it by exact construction as follow. we use the length $\sqrt{lb}$ so obtained in following construction to cut the $L$ shaped area as shown in figure from the hollow square instead of using algebraic equations. In Figure 2 draw an

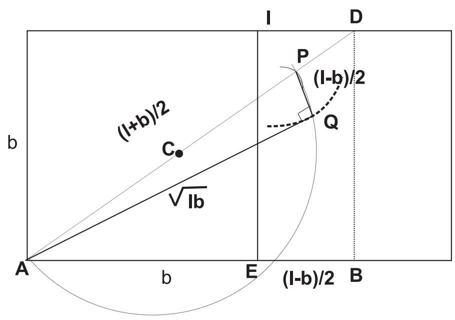


**Figure 6.**

arc of radius $AB = \frac{l+b}{2}$ centered at $A$ on $AD$. Find the center of $AD$ say $C$ and draw a semicircle centered at $C$ on $AD$. Let $P$ be the point of intersection of semicircle with $AD$. Now draw an arc of radius $EB = \frac{l-b}{2}$ centered at $P$, suppose it intersects the semicircle at $Q$. As a result we get a right angle triangle $AQP$ with $Q$ as a right angle. By application of Pythagarous Theorem $AQ = \sqrt{lb}$. From the Figure 5B area between the squares is

$$\Big(\frac{l+b}{2}\Big)^2 - \Big(\frac{l-b}{2}\Big)^2 = lb$$

Now from the construction of $\sqrt{lb}$ from the Figure 6 let us construct a point $P$ on $AB$ at $\sqrt{lb}$ from $A$ and a point $Q$ on $DC$ at $\sqrt{lb}$ distance from $D$. Complete the square $NMQD$ as shown in the Figure 7. This the required square.

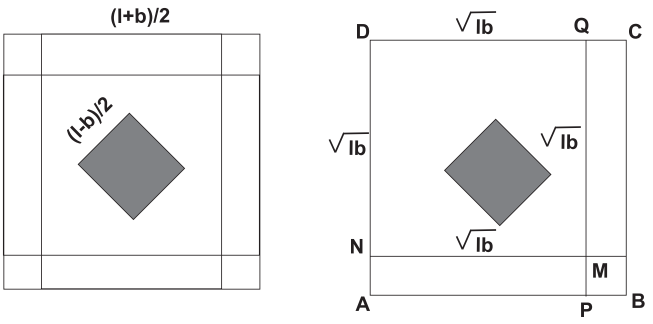


Figure 7.

## 4. Further Scope

The same solution based on the geometrical construction can further can be conformed using coordinate geometry. This will pause the following basic questions.

1. Choice of the rectangle place in the coordinate plane.
2. Scaling of basic lengths.
3. Obtaining coordinates of all intermediate points.
4. Obtaining equations of intermediate and final constructions.

## 5. Conclusion

The students can understand various fundamental concepts like geometry, set theory, series and means, limit and optimizations. Summarily, it can be stated that mathematically this problem is a problem to find the geometric mean of two real positive numbers. Interpretation wise geometric mean is the side of a square whose area is constructed from a rectangle without wastage of area. Learned community often avoid such problems under the assumption of treating them as simple problems.

## 6. References

[1] The textbooks of basic mathematics.