Synthesis of Four Bar Mechanism for Polynomial Function Generation by Complex Algebra

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Abstract—General solutions for determining link lengths of four bar mechanism for function generation where function is polynomial equation up to second order. Different methods are available for synthesis of four bar linkages and amongst this Complex Algebra method has its useful characteristics in the synthesis problems. In precision point approach, the desired motion characteristics are achieved at precision points only and at all other points there is structural error. The generalized formulation thus obtained is coded and results are obtained using MATLAB 7.6. It provides synthesis procedure making use of Chebychev Spacing.

Keywords- Kinematic Dyad Synthesis, Function Generation, Structural Error, Transmission Angle;

I. INTRODUCTION

The important and complementary area called kinematics synthesis, where mechanism is created to meet certain motion specification is touched upon only by a consideration of the simple aspect of planar linkage synthesis. Four-bar linkage has various applications like automobiles, endolite prosthetic knee, and mechanism for steam engines. The criteria for the design of mechanism are low fluctuation of input torque, compact in size and links proportion, good in force transmission, low periodic bearing loads, less vibrations, less wear, optimum transmission angle and higher harmonics, space requirement. The four-bar linkage has a long history in both the theoretical kinematics literature as well as a variety of applications. The transmission angle is one of the important design criteria to be considered for the effective force/motion transmission by a mechanism was pointed by Bali [2].

II. SYNTHESIS OF MECHANISM

A frequent requirement in design is that of causing an output member to rotate, oscillate or reciprocate according to the specified function of time or function of the input motion; this is called the function generation. When a mechanism is designed to generate a given function or trace a given curve, it is not possible in general to obtain a mathematically exact solution but that the mechanism fits the function or curve at only a finite number of points, the accuracy points. The number of these accuracy points is equal to the number of fixed parameters that may be used in the synthesis. The problem considered here is that of spacing the accuracy points within the interval of function generation to minimize the errors between accuracy points. Given multiple sets of precision points generated by function, the objective is to synthesis a mechanism that can trace each set of precision points. Consider the function f(x) to be approximated in a given interval of variation of x by means of a mechanism which generates function F(x; D₁, D₂,..,Dₙ) where D₁,...,Dₙ are the values of the n design parameters in the linkage. The difference between these two functions is the structural error. f(x)-F(x, D₁, D₂,...,Dₙ) The general appearance of the structural error when plotted against x.
Transmission angle (µ) for four bar mechanism is the angle between coupler and the follower as shown in figure. Where, β is the angle between coupler and driver as shown in figure 1[4].

\[ \mu = \cos^{-1} \left( \frac{l_3^2 + l_2^2 - l_1^2 - l_2^2 + 2l_1l_2 \cos \theta}{2l_3l_4} \right) \]  \hspace{1cm} (1)

\[ \beta = \cos^{-1} \left( \frac{l_2^2 + l_3^2 - l_1^2 - l_4^2 + 2l_1l_4 \cos \theta}{2l_2l_3} \right) \]  \hspace{1cm} (2)

The maximum and minimum values are shown in figure 2. The minimum value of transmission occurs at 0 degree of input angle. The corresponding values can be obtained by using equation 3. Mechanical Advantage is directly proportional to the sin of Transmission angle, and inversely proportional to sinβ, When sinβ = 0, The Mechanical Advantage = ∞. Thus at such a position only a small input torque is necessary to overcome a large output torque load. This is the case when the driver is directly in line with the coupler is said to be Toggle position. As Transmission Angle become small Mechanical Advantage decreases, and even small amount of friction will cause the mechanism to lock or jam [4].

\[ \cos \mu_{\text{min-vmax}} = \frac{l_3^2 + l_2^2 - l_1^2 - l_2^2 + l_1l_2}{2l_3l_4} \]  \hspace{1cm} (3)

III. MATHEMATICAL FORMULATION

For function generation Formulation:

Initially taking polynomial equation as function, \( Y = f(x) \),

\[ Y = a_0 + a_1x + a_2x^2 \]  \hspace{1cm} (4)

Where \( a_0, a_1, a_2 \) are constants of polynomial equation. Using chebychev spacing derives equation which gives relation between constants of coefficient of polynomial equation and link lengths of four bar mechanism.

From \( n \) accuracy points, by using chebychev spacing, for given range \( X_0 < X < X_{n+1} \), precision points are obtained.

If \( \theta \) is input crank angle, \( \varphi \) is output follower angle and ranges from 0° to 360°. If \( f \) and \( s \) consider as final and start values then, Range of \( \theta = \theta_f - \theta_s \), Range of \( \varphi = \varphi_f - \varphi_s \).

By assuming linear relationship between \( x \) and \( \theta \), from equation (5) values of \( \theta \) and \( \varphi \) are obtained.

\[ \frac{x_f - x_s}{\theta_f - \theta_s} = \frac{\varphi_f - \varphi_s}{\theta_f - \theta_s} \]  \hspace{1cm} (5)

Complex Number Modeling in Kinematic Synthesis

The great majority of planar linkage may be thought of as combination of vector pair called ‘Dyads’ [3]. The equation of closure for the four bar links if \( Z_1, Z_2, Z_3, Z_4 \) are assuming as link lengths of four bar mechanism then in the first position equation will be,

\[ Z_2 + Z_3 + Z_4 - Z_1 = 0 \]  \hspace{1cm} (6)

In case of function generation, only angular relations between the linkages of the mechanism will be considered, so it can be set,

\[ Z_1 = 1 \text{ then}, \]

\[ Z_2 + Z_3 + Z_4 - 1 = 0 \]  \hspace{1cm} (7)

In polar notation

\[ Z_2 e^{i\theta_2} + Z_3 e^{i\theta_2} + Z_4 e^{i\theta_4} - 1 = 0 \]  \hspace{1cm} (8)

The three equations for three precision points,

\[ Z_2 + Z_3 + Z_4 = 1 \]

\[ Z_2 e^{i\theta_2} + Z_3 e^{i\theta_2} + Z_4 e^{i\theta_2} = 1 \]

\[ Z_2 e^{i\theta_3} + Z_3 e^{i\theta_3} + Z_4 e^{i\theta_3} = 1 \]
Considering the three precision positions

θ₁ = 36.028°, \ θ₂ = 75.00°, \ θ₃ = 113.97°,

ϕ₁ = 54.15°, \ ϕ₂ = 87.50°, \ ϕ₃ = 132.09°.

Link lengths are: \ [Z₁, Z₂, Z₃, Z₄] = [1, 0.995, 0.78, 0.96]

IV. RESULTS AND DISCUSSION

The start and final range of range of X will be taken as 1 and 3 respectively then by use of Chebychev formula for three precision points, function generator values can be obtained. By assuming range of input and output the following result table will be obtained.

TABLE I. RESULT TABLE

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Precision Points (X) Values</th>
<th>Function Generator (Y) Values</th>
<th>Input Angel (θ)</th>
<th>Output Angle (ϕ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.1340</td>
<td>4.5538</td>
<td>36.028°</td>
<td>54.15°</td>
</tr>
<tr>
<td>2</td>
<td>2.00</td>
<td>9.00</td>
<td>75.00°</td>
<td>87.50°</td>
</tr>
<tr>
<td>3</td>
<td>2.866</td>
<td>14.946</td>
<td>113.97°</td>
<td>132.09°</td>
</tr>
</tbody>
</table>

The generalized solutions for getting comparison between input angle and transmission angle, bita angle, mechanical advantage and generalize code for getting link lengths for function generation and finding structural error are obtained. The basic procedure for the numerical solution is as follows:

a. Calculate all values of x from chebychev spacing.
b. Take values of a₀, a₁, a₂.
c. Calculate y values by substituting x values in polynomial equation.
d. Take the values of θᵢ, θᵢ, ϕᵢ, and ϕᵢ.
e. Calculate θᵢ and ϕᵢ by using linearity.
f. Prepare dyad equations for three precision points.
g. Evaluate link lengths from dyad equations.

Taking various combination of input – output ranges and different constants of co-efficient of polynomial equations, synthesis can be done from generalize code in MATLAB.

Example

For function \ y = 1 + 2x + x², \ start rage of \ xᵢ = 1 \ and final range of \ xᵢ = 3, \ number of precision point \ n = 3. \ Value of constants of polynomial equation \ a₀ = 1, \ a₁ = 2 \ and \ a₂ = 1, \ θᵢ = 30°, \ θᵢ = 120°. \ Φᵢ = 50° \ and \ Φᵢ = 140°, \ fixed link length \ r₁ = 1. \
Figure 8 shows a linear relationship between \( x \) values and \( \theta \).

Figure 5 shows a linear relationship between \( y \) values and \( \phi \).

Figure 6 shows a parabolic relationship between \( \theta \) and \( \phi \).

Figure 7 shows a parabolic relationship between \( x \) and \( y \).

Figure 8 shows a plot for Output Angle vs Input Angle.

Figure 9 shows a plot for Transmission Angle and Mechanical Advantage.

It can be concluded that as Transmission Angle increases, Mechanical Advantage increases up to input Angle becomes 90°, then it will be decreases. Figure 10 shows a plot for Input Angle and Bita angle Minimum value of \( \beta \) is 35° at 180° crank angle and maximum value of 60° at 90° crank angle.

Figure 11 shows the plot for input crank angle vs transmission angle. It can be concluded that minimum value of transmission angle is 15.66° at input crank angle 0° that of maximum value 90° at input 180°.

V. STRUCTURAL ERROR

A mechanism has only a limited number of designed parameters, a few link lengths, starting values for the input and output so a linkage synthesis problem usually has no exact solutions over its entire range of travel. So structural error is defined as the theoretical difference between the function prescribed by the synthesized linkage and the function originally prescribed.

Example

For function \( y = 1 + 2x + x^2 \), start range of \( x = 1 \) and final range of \( x = 3 \), number of precision point \( n = 10 \). Value of constants of polynomial equation \( a_0 = 1, a_1 = 2 \) and \( a_2 = 1 \), \( \theta_s = 30°, \theta_f = 120°, \phi_s = 50° \) and \( \phi_f = 140° \), \( l_1 = 1, l_2 = 0.99, l_3 = 78, l_4 = 99 \), find the structural error for generated function.
TABLE II. RESULT TABLE FOR STRUCTURAL ERROR

<table>
<thead>
<tr>
<th>Sr. No</th>
<th>Precision Points (X) Values</th>
<th>Input Angle (θ)</th>
<th>Output Angle (φ)</th>
<th>Y coordinate</th>
<th>Y coordinate</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0123</td>
<td>30.554°</td>
<td>52.5102°</td>
<td>4.0494</td>
<td>4.2769</td>
<td>-5.61812</td>
</tr>
<tr>
<td>2</td>
<td>1.109</td>
<td>34.9047°</td>
<td>54.3223°</td>
<td>4.4479</td>
<td>4.5484</td>
<td>-2.25949</td>
</tr>
<tr>
<td>3</td>
<td>1.2929</td>
<td>43.1802°</td>
<td>59.3644°</td>
<td>5.2574</td>
<td>5.2649</td>
<td>-0.14266</td>
</tr>
<tr>
<td>4</td>
<td>1.546</td>
<td>54.5704°</td>
<td>68.1857°</td>
<td>6.4822</td>
<td>6.4913</td>
<td>-0.14038</td>
</tr>
<tr>
<td>5</td>
<td>1.8436</td>
<td>67.9604°</td>
<td>80.0294°</td>
<td>8.0859</td>
<td>8.1275</td>
<td>-0.51448</td>
</tr>
<tr>
<td>6</td>
<td>2.1564</td>
<td>82.0396°</td>
<td>93.5287°</td>
<td>9.9631</td>
<td>9.9968</td>
<td>-0.33825</td>
</tr>
<tr>
<td>7</td>
<td>2.454</td>
<td>95.4296°</td>
<td>107.196°</td>
<td>11.9301</td>
<td>11.9106</td>
<td>0.163452</td>
</tr>
<tr>
<td>8</td>
<td>2.7071</td>
<td>106.82°</td>
<td>119.617°</td>
<td>13.7426</td>
<td>13.6894</td>
<td>0.321628</td>
</tr>
<tr>
<td>9</td>
<td>2.891</td>
<td>115.095°</td>
<td>129.444°</td>
<td>15.1399</td>
<td>15.2106</td>
<td>-0.46698</td>
</tr>
<tr>
<td>10</td>
<td>2.9877</td>
<td>119.446°</td>
<td>135.147°</td>
<td>15.9017</td>
<td>16.2379</td>
<td>-2.11424</td>
</tr>
</tbody>
</table>

Figure 13. Structural Error

Figure 14. Plot for x range v/s Structural Error.

It can be concluded from graph error is more at extreme position as shown in table 2. Figure 14 shows structural error, and figure 12 shows plot for x range v/s % structural error.

VI. CONCLUSION

The general solution presented is very much useful to study of Analytical Synthesis of four bar mechanism for function generation with using chebychev spacing and Dyads Synthesis, where function is polynomial equation up to second order. Dyad methods give more precise results as it can take more number of precision points. Function gives parabolic curve. The synthesis of four bar mechanism for function generation can be understood. Generated function satisfies the Grashof’s condition, and gives fully rotating mechanism. By this optimum transmission angle will be obtained, and also structural error is not more than 3 to 4%. The maximum of minimum Transmission angle is 15.66° in Dyad synthesis, which is less than 45° [5], which satisfies the criterion for maximum mechanical advantage.

REFERENCES

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[6] MATLAB help